Introduction

This lesson covers dividing integer powers. It assumes students are familiar with exponents and multiplying powers. This lesson does not cover negative exponents explicitly, however.

Standards Assessed

California Content Standards for Algebra I – Grades 8-12

1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

Dividing Integer Powers

Simplifying quotients of powers can be as easy as cancelling factors. For example,

$$\frac{2^7}{2^4} = \frac{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2\cdot 2} = \frac{2\cdot 2\cdot 2\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}} = 2\cdot 2\cdot 2 = 2^3.$$

Notice that the number of 2s in the numerator is decreased by the number of 2s in the denominator (7 factors decreased by 4 equals 3 remaining).

Here is a harder example.

$$\frac{5^{36}}{5^{32}} \;=\; ???$$

If we had time to write down all the factors, we would see that the number of factors of 5 in the numerator (36) is decreased by the number of factors in the denominator (32), leaving $36 - 32 = 4$ factors of 5 remaining. That is,

$$\frac{5^{36}}{5^{32}} = \frac{5\cdot 5\cdot 5\cdot 5\cdot \overbrace{\cancel{5}\cdot \cancel{5}\cdot \cdots \cdot \cancel{5}}^{32}}{\underbrace{\cancel{5}\cdot \cancel{5}\cdot \cdots \cdot \cancel{5}}_{32}} = 5^{36-32} = 5^4$$

Question. What is

$$\frac{11^{100}}{11^{90}} \;=\; ?$$

Practice. Simplify the following.

$$\frac{3^3}{3^2} = \frac{3\cdot 3\cdot 3}{3\cdot 3} = \frac{3\cdot \cancel{3}\cdot \cancel{3}}{\cancel{3}\cdot \cancel{3}} = 3$$

$$\frac{10^5}{10^4} =$$

$$\frac{8^7}{8^3} =$$

$$\frac{6^{13}}{6^{12}} =$$

So far we have only divided monomials where the numerator has more factors than the denominator. Here is a different example.

$$\frac{3^3}{3^5} = \frac{3\cdot 3\cdot 3}{3\cdot 3\cdot 3\cdot 3\cdot 3} = \frac{\cancel{3}\cdot \cancel{3}\cdot \cancel{3}}{\cancel{3}\cdot \cancel{3}\cdot \cancel{3}\cdot 3\cdot 3} = \frac{1}{3^2}$$

Notice now that the number of factors in the denominator is decreased by the number in the numerator (the opposite of what we did before). The numerator is 1 because any number divided by itself is 1.

A Rule for Dividing Powers

The techniques we just used suggest that if x is not zero and a is greater than b, then

$$\frac{x^a}{x^b} = x^{a-b}$$

On the other hand, if b is greater than a, then

$$\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$$

Question. What do we get if a and b are equal?

More Practice.

$$\frac{4^3}{4^2} = \frac{4\cdot 4\cdot 4}{4\cdot 4} = \frac{4\cdot \cancel{4}\cdot \cancel{4}}{\cancel{4}\cdot \cancel{4}} = 4$$

$$\frac{10^4}{10^5} =$$

$$\frac{8^3}{8^7} =$$

Now let's make things a little more interesting. Consider the following.

$$\frac{2^6 \cdot 3^{10} \cdot 5^8}{2^4 \cdot 3^3 \cdot 5^2} \; = \; ???$$

The trick is to divide like factors. So

$$\frac{2^6 \cdot 3^{10} \cdot 5^8}{2^4 \cdot 3^3 \cdot 5^2} = \frac{2^6}{2^4} \cdot \frac{3^{10}}{3^3} \cdot \frac{5^8}{5^2} = 2^{6-4} \cdot 3^{10-3} \cdot 5^{8-2} = 2^2 \cdot 3^7 \cdot 5^6.$$

Mixing things up a bit, try

$$\frac{2^6 \cdot 3^3 \cdot 5^8}{2^4 \cdot 3^{10} \cdot 5^2} \; = \; ???$$

Notice that the exponent of 3 in the numerator is smaller than the exponent of three in the denominator, so we just subtract in the reverse order.

$$\frac{2^6 \cdot 3^3 \cdot 5^8}{2^4 \cdot 3^{10} \cdot 5^2} = \frac{2^6}{2^4} \cdot \frac{3^3}{3^{10}} \cdot \frac{5^8}{5^2} = \frac{2^{6-4} \cdot 5^{8-2}}{3^{10-3}} = \frac{2^2 \cdot 5^6}{3^7}.$$

Even More Practice.

$$\frac{11^{11}}{11^{11}} =$$

$$\frac{10^{11} \cdot 11^{10}}{11^{11} \cdot 10^{10}} =$$

$$\frac{2^3 \cdot 3^4 \cdot 4^5}{2^2 \cdot 3^2 \cdot 4^8} =$$